Backlash

...a purposeful error

Modeling of Backlash in Worm Gear using Iconic Diagram

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Engineering is all about making systems with accuracy. Engineers study, research and learn to avoid errors in system development. Especially when its about developing a mechanical system, the concern is to make parts with high degree of accuracy and no chances of errors.
But Backlash is probably one of its kind error which is purposefully kept in mechanical parts. Why…?

In this article we would study, model and analyze backlash effect which is common in drive trains. Worm Gear is selected to show Backlash. Model (fig 3) represents a worm gear with spindle (also referred to as worm) and wheel (also referred to as gear). Between spindle and wheel, backlash and friction is modeled.

Induced Vibrations has been shown with the help of 20-Sim, which is a dynamic modeling and simulation package available from Controllab, Netherlands.

About Backlash:
In mechanical engineering, **backlash** is purposeful clearance between mating components, sometimes described as the amount of lost motion due to clearance or slackness when movement is reversed and contact is re-established. For example, in a pair of gears, backlash is the amount of clearance between mated gear teeth. This gap means that when a gear-train is reversed the driving gear must be turned a short distance before all the driven gears start to rotate. At low power outputs, backlash results in inaccurate calculation from the small errors introduced at each change of direction; at large power outputs backlash sends shocks through the whole system and can damage teeth and other components.

Backlash is an unavoidable property of all reversing mechanical couplings, and may or may not be desirable, depending on the application. Gears with backlash are common in automatic and manual transmissions and differentials. Some backlash is required to allow for lubrication, manufacturing errors, deflection under load and differential expansion between the gears and the housing.

Backlash is created when the tooth thickness of either gear is less than the tooth thickness of an ideal gear, or the zero backlash tooth thickness. Additional backlash is created when the operating center distance of the gear pair is less than that for two ideal gears. The total backlash is defined as:

$$b = bt + bc$$

Where:
b = total backlash,
bt = backlash due to tooth thickness modifications
bc = backlash due to operating center distance modifications

Backlash due to tooth thickness changes is typically measured along the pitch circle and is defined by:

$$bt = ti - ta$$

Where:
ti : tooth thickness on the pitch circle for ideal gearing
Backlash, measured on the pitch circle, due to operating center modifications is defined by:

$$b_c = 2(\Delta c) \tan\Phi$$

Where:
Δc : Difference between actual and ideal operating center distances

Φ : Pressure angle

Standard practice is to make allowance for half the backlash in the tooth thickness of each gear. However, there are applications where this may not be advisable. For example, if the pinion (the smaller of the two gears) is too small or has few teeth, the engineer may not modify the pinion teeth and instead, take the total backlash out of the gear teeth.

Analysis:
The spindle and wheel have one or more pair of teeth contacting each other. The effective point of contact is called the pitch point ***P***. At the pitch point a frame is defined, with an x- and y-direction. During rotation of the wheel and spindle, the teeth will experience a normal force *Fn* in the x-direction and a friction force *Ff* in the y-direction.

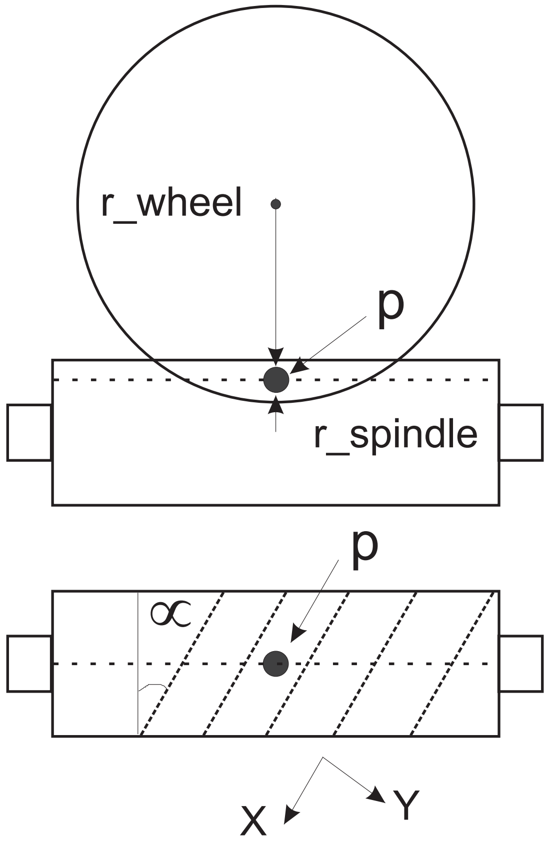


Figure: 1 Worm gear from design point of view

Between each pair of teeth, a clearance exists (y-direction), causing backlash. This effect is time-dependent, because of the tooth pairs coming in and out of contact. During normal operation this is a high frequency effect, which is filtered out by the damping of the construction. We therefore only used an **average backlash** at the pitch point. Using the same reasoning as backlash, an **average friction** is used at the pitch point.

To define both backlash and friction **a frame is used at the pitch point** (figure 1) to indicate velocities. The velocity in x-direction is the sliding velocity between the teeth and used to calculate the friction force. The position in y-direction is the position in the clearance and used to calculate the backlash force.

Involved Mathematics:
The backlash is described by the standard formula that is also used in the Backlash model. The normal force is modeled by a spring-damper system. Inside the clearance a low damping and stiffness is used (*k1* and *d1*), while a high stiffness and damping (*k2* and *d2*) is used at both ends of the clearance. This yields a normal force *Fn* of:

$$p.F = 0.5k_2\left(x - \sqrt{(-x - s/2) + (ep.s)^2} + x + \sqrt{(-x - s/2) + (ep.s)^2}\right)$$
$$+ k_1.x + d_{1or2}.p.v$$

Friction is described as static plus coulomb plus viscous plus Stribeck friction:

p.F = Fn *((mu_c + (mu_st*abs(tanh(slope*p.v)) - mu_c) * exp(-((p.v / v_st)^2))) * sign(p.v)+ mu_v * p.v);

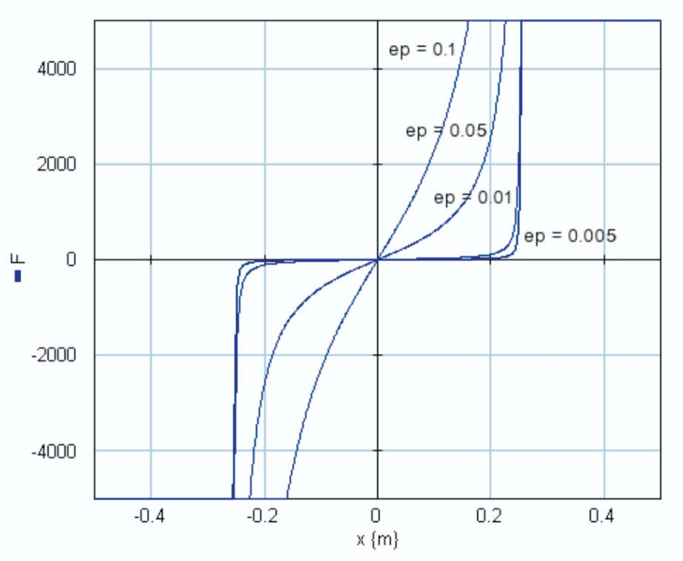
with:

Fn	: the normal force (given by the backlash formula)
Mu_s	: the static friction coefficient
Mu_v	: the viscous friction coefficient
Mu_c	: the coulomb friction coefficient
Slope	: the steepness of the coulomb and static friction curve.
V_st	: the characteristic Stribeck velocity.

The model represents backlash by a spring damper system. Inside and outside the play, spring and damping can be set separately. Discontinuities are avoided by adding a round off. The model can have a force out causality. The port *p* of this model has separate high and low terminals. The equations are:

p.F = p_high.F = p_low.F
p.v = p_high.v - p_low.v

With x, the position within the play, real backlash behavior is



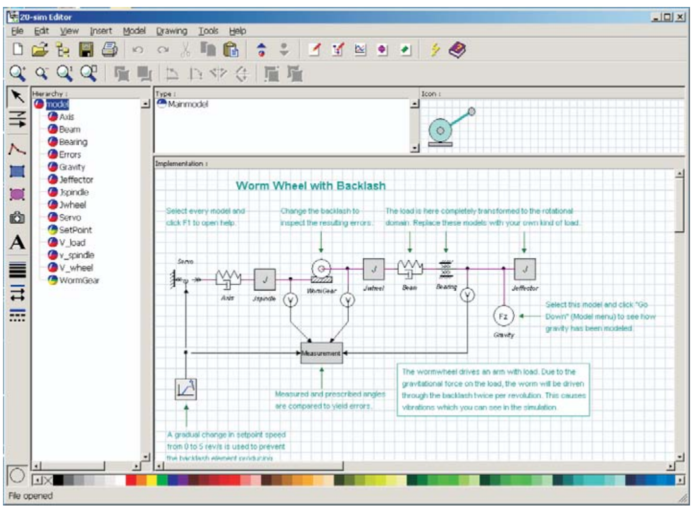
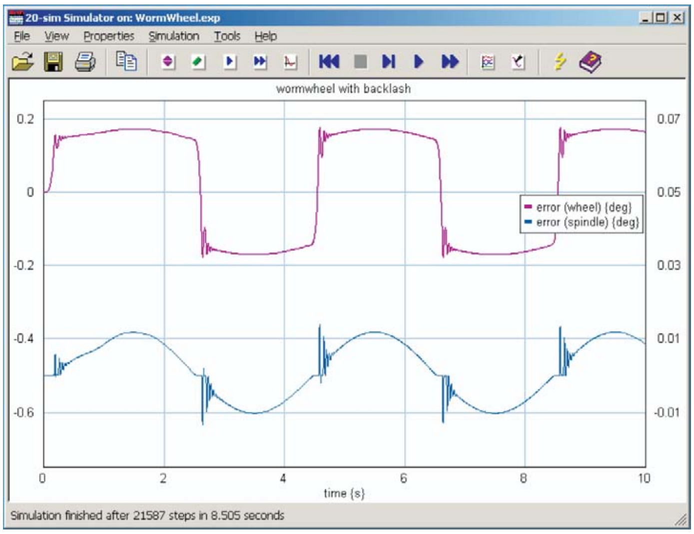


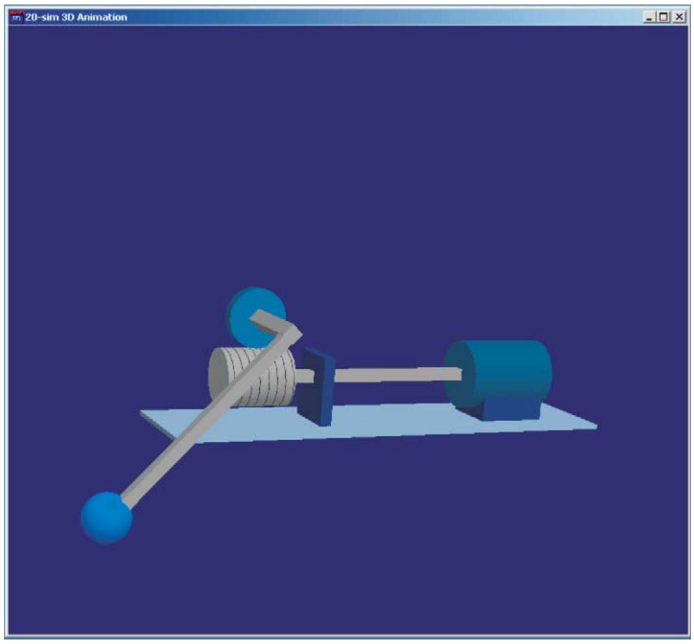
Figure: 3 Worm Gear System model

obtained by choosing low stiffness and damping values inside the play and choosing high stiffness and damping values outside the play.

The parameter 'ep' determines the smoothness of the force curve that is obtained. A larger value (> 0.01) makes the force change gradually when the position reaches the play boundaries. A smaller value (< 0.001) makes the force change abruptly. This is shown in the fig. 2 with parametric sweep facility of time domain toolbox available in 20-Sim. A good starting value for ep is 1e-4.

Modeling in 20-Sim:
The worm gear model is shown in fig. 3. This is modeled by iconic diagram supported by 20-Sim. These *icons* represent physical elements with underlying sets of equations describing their behavior. Icons are connected together to form causal relationship forming a system. The model consists of an ideal motor that yields a prescribed rotary motion. This prescribed motion is achieved with motion profile wizard available in *Mechatronics toolbox* of 20-Sim. This wizard outputs position, velocity and acceleration curves using different polynomials suitable of any kind of realizable motion with mechanisms. A gradual change in set point speed from 0 to 5 rev/s is used to prevent the backlash element producing extreme start-up vibrations. The rotary motion is converted to a rotation of a lever. The load is here completely transformed to the rotational domain. Velocity of spindle and wheel is sensed by velocity sensor, which is given to measurement block where time integral of velocity is evaluated to give position. Each time the lever passes the right-up position (either hanging or standing) it is forcing the backlash to traverse the play zone. As lever moves up gravitational force acts on it which models with gravity block. The backlash represents here the freedom of movement that the gearwheel teeth experience between the spindle teeth. Although the chosen backlash is very small (100 um), due to small radius of the worm wheel (20 mm) the freedom of movement of the lever is noticeable (0.3 degree).





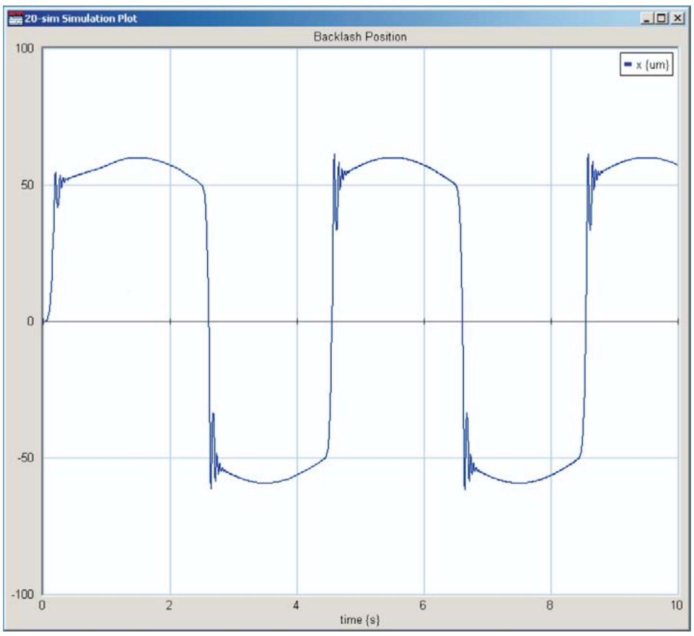


Figure: 5 Backlash Position

Simulation Results:

The following figure shows simulation results obtained with Vode Adams integration techniques with absolute and relative tolerances set to 1e-006.

Fig 4 shows position error of the input shaft (spindle) and lever (wheel)

.At the edge transitions simulation results has shown damped periodic oscillations due to presence of backlash. Model Analysis with FFT has shown fundamental component 1.255 rad/sec. Backlash position is shown in fig 5. Curve shows amplitude reaching to ±50 μm which is confined with 100 μm specified in model.

Conclusion:
Worm wheel assembly is modeled and simulated. Backlash has caused *vibrations* in motion which are shown in result with aid of 20-Sim. Increasing backlash has shown increase in positional errors. Self locking of worm gear can be verified by increasing friction force.